Ph2a Online @ Caltech **Homework-1**

Vibrations and Waves

Reading: Read Chapter 1 of the textbook, on simple harmonic motion.

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1. Read the brief note on effective problem solving, appended below. Try to apply the suggestions in this note to the following problems, and all your problem sets (not only this course!).

2. You are given simple harmonic motion described by:

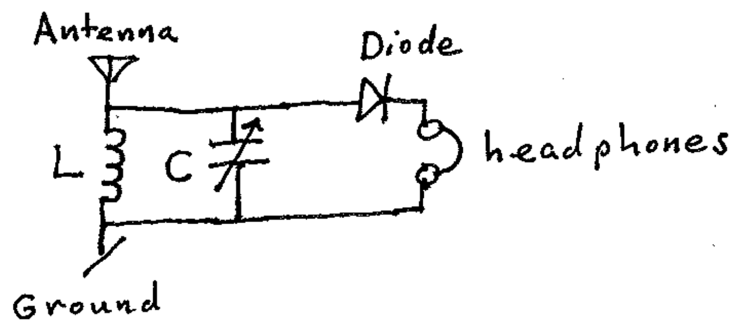
$$x(t) = A\cos(\omega t + \phi), \tag{1}$$

with $A = 10$ cm, $\omega = \pi/4$ radians/s, and $\phi = \pi/4$. Draw graphs showing the time dependence of

   (a) Position
   (b) Velocity
   (c) Acceleration

Show at least two full cycles and include $t = 0$. You may make your graphs by hand or with a program such as Matlab, Mathematica, etc, but in any case be careful to label your axes and include units. Make sure it is clear to the grader where the zeros and minima/maxima are.

3. In ancient times, long before the internet and smartphones, students would get their entertainment over the air by listening to AM radio. This was also before credit cards, and to save money we would make our own receivers. Here is a typical circuit:

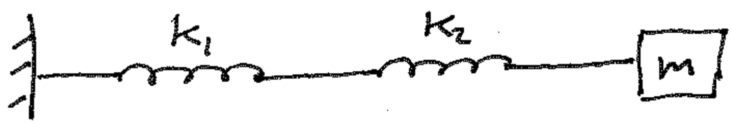


Note the environmental friendliness of this, no batteries required! We were so virtuous in those days.

Well, there are still some AM stations around, for example, if you want news, you might listen to KNX 1070. The 1070 means the carrier frequency is 1070 kHz. Suppose you have a 33 μH inductor and want to be able to tune in KNX. What value do you need for the capacitor? Note that the point is to "tune" the oscillator frequency to the desired radio frequency. We'll learn more about why this is a good idea as we go along.

4. Journey to the center of the earth: Imagine that there is a (straight) hole all the way through the earth (passing through the center). Ignoring certain practicalities such as temperature, imagine further that you step into this hole from its rim. Do you execute simple harmonic motion? If so, what is your frequency in Hz? You may assume that you step very precisely and don't bounce off any walls.

5. Consider a mass on two springs in series as shown in the diagram:



(a) In terms of k_1, k_2, and m, what is the natural frequency of this oscillator? Note that it is not sufficient to look up the answer somewhere and copy it – you must derive it for yourself (and show your work).

(b) If the frequency due to spring one alone is 1 Hz and spring two alone is 2 Hz, what is the frequency when the two springs are in series?

...problem solving note appended below, on the next page.

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Note on Effective Problem Solving

A goal for this course is to help you to become a better problem-solver. The graders will look for "good" solutions that have characteristics along the lines illustrated in the example here. You may lose points even if you get the "right" answer if it is not clear, if you haven't stated assumptions clearly, or if you have not demonstrated a logical approach.

We'll start with some tips:

1. Don't rush to the text for "The Formula". You'll learn much more if you do some thinking for yourself.
2. Draw a picture! This helps to collect your thoughts and make sure you understand the problem.
3. Try to develop some intuition for how you think the system will behave, or for what the answer ought to be. You may discover that you are wrong, but it can help to guide your approach, and you can modify your thinking as you go along.
4. Define relevant quantities. Before writing down numbers, define some symbols, which will help you keep track of what you mean. It also means you will have derived something more generally useful.
5. Give some thought to your choice of symbols. In may cases, you'll just use whatever we use in lecture and the book. But try to choose symbols that are meaningful to you and not a source of confusion.
6. You will likely want/need to set up a coordinate system. Try to choose something that makes the problem look "simple". This might mean using an angle rather than a cartesian axis, for example. Symmetries can often be taken advantage of. Choose the origin for simplicity if you can.
7. Think about and state assumptions that you think are implicit in the problem statement. We won't necessarily state all of these when giving the problem. This reflects real-world problems, where part of understanding and attacking a problem is to identify what the problem is.
8. Finally, make sure the units are sensible. If one side of an equation evaluates to a length, the other should as well. It is remarkable how often people neglect this simple sanity check!

Now let's consider attacking a sample homework problem:

Problem: Given a 1 kg mass on a spring, with spring constant $k = 1$ N/m, what is the natural frequency of oscillation?

Here are two attempts at a solution:

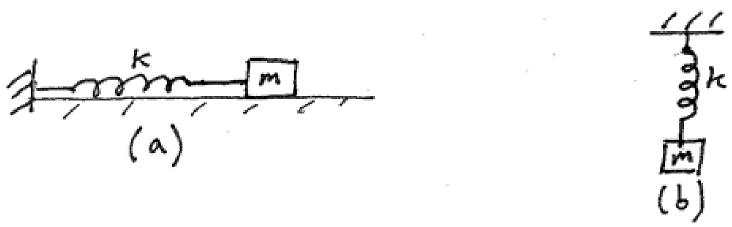
The solution that makes your instructor very sad:

$$\sqrt{1/1} = 1 \text{ s}^{-1}.$$

Comments: I expect the grader to seriously ding you if you turn in a solution such as this, even if the answer is nominally "correct". You have not explained what you are doing. The problem statement here is quite vague, and you have not said what assumptions you are making. Have you thought about whether the problem is horizontal or a mass hanging from a spring under gravity? You have not been very clear about what you are calculating. You give units of s^{-1}, but since we are talking about frequency, do you mean Hz? Or is it radians/s?

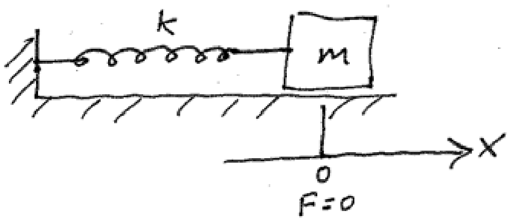
The gold star solution:

Hmm, this is pretty vague, what does it mean? Is it a horizontal spring, like picture (a), or a vertical one, like picture (b)? In either case, I'll assume we are on the Hooke's law region of the spring, since that is implicit in asking for the natural frequency.



Fortunately, the natural frequency will be the same either way, since only the equlibrium length of the spring is altered between these two possiblities. I'll go with (a) to keep it simple. I'm also going to assume that the mass slides without friction, since this is implicit in finding the natural frequency. I'll also assume the spring is massless, since we don't have any further information about it.

Now let x be the horizontal coordinate along the direction of motion, with origin at the equilibrium position:



We have a restoring force on the mass of $-kx$, or

$$F = ma = -kx,$$

where $a = d^2x/dt^2$ is the acceleration. Thus, the differential equation of motion is:

$$m\frac{d^2x}{dt^2} = -kx.$$

The general solution may be written in the form

$$x(t) = A \sin \sqrt{\frac{k}{m}} t + B \cos \sqrt{\frac{k}{m}} t,$$

where A and B are constants of integration that we won't need to worry about here, since they don't affect the oscillation frequency.

A full oscillation of the system thus occurs when $\sqrt{k/m}t = 2\pi$. That is, the period of oscillation is $T = 2\pi/\sqrt{k/m}$, and hence the natural frequency of the oscillator is:

$$f = 1/T = \sqrt{k/m}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{1 \text{ N/m}}{1 \text{ kg}}} = 0.159\ldots \text{ Hz}.$$

Alternatively, the natural frequency can be expressed as

$$\omega = 2\pi f = 1 \text{ radians/s}.$$

Comment: I have chosen one particular path to solution here, many other paths are also fine, as long as your solution is clearly developed.