Reading: Read Chapter 2 of the textbook, on damped harmonic motion, sections 3.1 and 3.2 on forced oscillations.

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6. A catenary is the curve made by a heavy chain suspended from fixed supports at both ends. The equation of a catenary is

$$y(x) = a\left(\cosh\frac{x}{a} - 1\right), \tag{1}$$

where x is a horizontal coordinate and y is a vertical coordinate. Suppose a mass m slides without friction along the catenary arc. What is the natural frequency of oscillation (under gravity) for small motion?

[Aside: derivation of the catenary shape is a nice illustration of variational calculus].

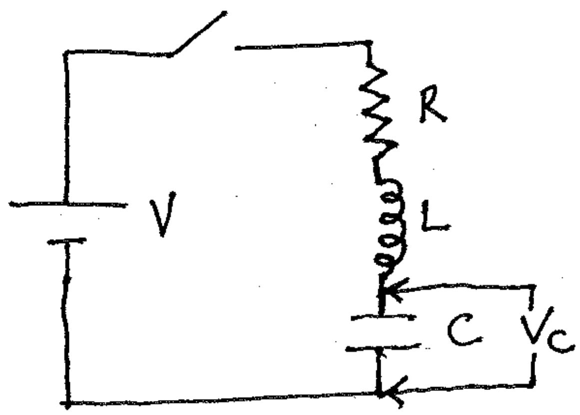
7. Suppose we have two pendulums (aka pendula), A and B, with identical geometry, but different masses, m_A and m_B. They are in air, so there is some (light) damping. It is observed that pendulum A damps to one-half its initial amplitude in 100 s, and pendulum B damps to one-half its initial amplitude in 1000 s. If $m_A = 9$ kg, what is m_B?

8. Critical damping:

   (a) Our discussion of the quality factor largely presumed the case of light damping, but we can use the definition we obtained for any damping. Thus, what is Q for the case of critical damping?

   (b) Suppose we have a critically damped system such as a mass on a spring. Let the natural frequency be $\omega_0 = 0.5$ radian/s. The system is released from a non-equilibrium position at time $t = 0$ with zero velocity. At what time does the system reach maximum speed?

9. Consider the circuit below:



The arrows with the designation "V_C" is just meant to indicate an instrument measuring the voltage across the capacitor, without affecting the operation of the circuit.

   (a) If $R = 500\Omega$, $L = 10$ mH, and $C = 1\ \mu$F, is the circuit lightly damped, critically damped, or heavily damped?

   (b) Suppose our figure has values of R, L, and C corresponding to a heavily damped oscillator. Suppose the switch is open for $t < 0$ and the voltage across each component is zero (except the battery). At $t = 0$ we close the switch. What is the voltage on the capacitor for $t > 0$? You may assume that the battery has zero impedance.

10. Consider again the circuit in problem 9. Suppose $\sqrt{1/LC} > R/2L$. The same initial conditions apply as in problem 9. For $t > 0$, what is the power dissipated in the resistor as a function of time? You may express your answer in terms of V, L, C, R or suitable functions of these that you define.
