Ph2a Online @ Caltech **Homework-3**
Vibrations and Waves

Reading: Finish reading Chapter 3 of the textbook, on forced oscillations. Read sections 4.1-4.2 on coupled oscillations.

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11. We mentioned that not all dissipative forces are proportional to velocity [Note that many physics texts, including ours, glibly ignore this important possibility!]. Let us investigate this briefly for a more realistic model of sliding frictional damping of a mass on a spring. Thus, we model friction as:

    - Static friction: If $\dot{x} = 0$ there is a frictional force opposing moving the mass, $F_s = \mu_s m$, where μ_s is the static coefficient of friction.
    - Kinetic friction: If $\dot{x} \neq 0$ there is a different frictional force opposing the motion, $F_k = \mu_k m$, where μ_k is the kinetic coefficient of friction. Note that there is no velocity dependence to this force other than the distinction between a stationary or moving mass. The static coefficient is larger than the kinetic – $\mu_s > \mu_k$.

    (a) Suppose that we pull the mass to position x_0, with $\dot{x}(0) = 0$, and release it. What do you think will happen?

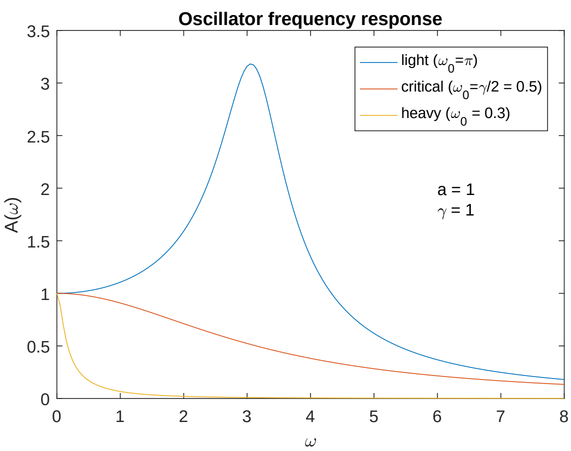
    (b) Does the motion stop in finite time? If so, what is the stopping condition? Note that our damped simple harmonic motion does not stop in finite time, although the amplitude decreases exponentially.

    (c) Write down the differential equation for the motion, and solve for $x(t)$. Hint: you may want to solve separately depending on whether the motion is to the left or to the right. When you solve the diffential equation does your answer make sense? If not, what is going on? Hint: Think about the turning points. I'm not looking for a final full solution to the motion, just an understanding of the motion.

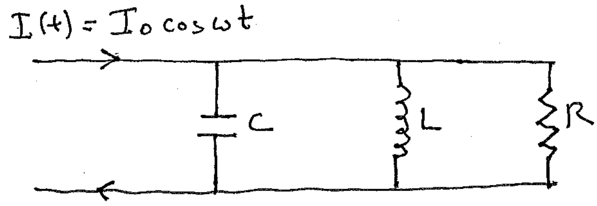
12. This is really the third part of problem 10, but I thought the algebra might be too much for one problem. Thus, consider problem 10 regarding the power dissipation in the circuit in problem 9. Suppose $\sqrt{1/LC} > R/2L$. The same initial conditions apply as in problem 9. What is the total energy ($t = \infty$) dissipated in the resistor? In the end you should have a simple expression that I hope will have some intuitive meaning for you. In case you need it, the power dissipated as a functon of time is

$$P(t) = R(CV\omega_0^2/\omega)^2 e^{-\gamma t} \sin^2 \omega t. \tag{1}$$

13. We looked at the frequency response for the steady state motion of a forced oscillator, for the three cases of light, critical, and heavy damping reproduced below:



Now make a similar graph showing the phase as a function of frequency for the three cases of light, critical, and heavy damping. I suggest you let a computer do the drawing, though a careful sketch is fine.



14. Consider the parallel RCL circuit below:

    (a) What is the natural frequency of this oscillator? What is the quality factor?

    (b) Considering the steady state solution, what is the peak voltage at resonance? Compare with what the peak voltage would be without the capacitor and inductor.
