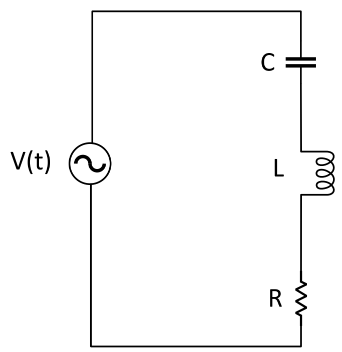
Reading: Finish reading Chapter 4 of the textbook, on coupled oscillations. Read the introduction to travelling waves on pages 105-109.

---

15. In lecture, we played with the LCR circuit below, with driving voltage

$$V(t) = V_0 \cos \omega t. \tag{1}$$



In particular, we made observations about how the phase of the voltage across the capacitor compared with the driving phase, as a function of driving frequency.

(a) Now make your postdiction for this and consider also the inductor and resistor: Draw a sketch of the phase difference between the voltage across each component and the driving voltage, as a function of the driving frequency. You can probably just do this with a hand sketch well enough to show the salient features (show the phases for all three components on the same graph). NOTE: This is a great place to use the exponential notation (e.g., write $V(t) = V_0 e^{i\omega t}$)! Please assume steady state conditions, that is, times long after any transient response has damped out.

(b) It will help with intuition to consider some limiting cases for the voltages across the components. Thus, what are the voltages across the capacitor, inductor, and resistor in the limit $\omega \to 0$? Likewise, what are the voltages across the capacitor, inductor, and resistor in the limit $\omega \to \infty$?

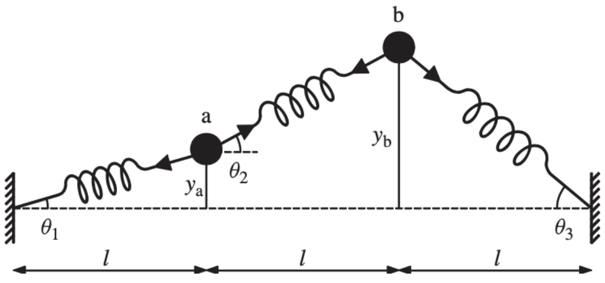
I hope that your answers to parts (a) and (b) will make sense and hang together for you.

16. Suppose you want to build the LCR circuit in the figure of problem 16 (maybe for a Ph 2a demonstration...), with $V_0 = 10$ V, $C = 10\ \mu$F, $L = 1$ mH, and $R = 40\ \Omega$. We want to run it at any frequency. You have a resistor rated for 2 watts. Do you think this is sufficient? Be sure to consider that the resistor doesn't dissipate energy instantaneously, so if the frequency is high compared to typical time scales for heat transport, it is the average power that is important.

17. In lecture we considered the problem with two identical pendula (length ℓ) coupled with a spring (spring constant k). Now consider the same problem except, instead of identical bob masses, with bob masses m_a and m_b. What are the frequencies of the normal modes? Does you answer agree with what you expect in the limit $m_a = m_b$.

18. Going back to two two identical pendula (length ℓ) coupled with a spring (spring constant k): Suppose we initially displace pendulum a by $x_a(0) = A$, with $x_b(0) = 0$. We start with everything at rest, $\dot{x}_a(0) = \dot{x}_b(0) = 0$. Will pendulum b ever reach $x_b = A$? If the answer is "it depends", under what conditions will this happen?

19. We saw in class that we could probably easily generalize our system of pendula coupled with springs to any number of pendula. Let's investigate the similar issue for a system of masses coupled by springs with *transverse* motion for the masses. For example, if we have a system with two masses and three springs, our setup looks like,



Now suppose we have such a system with n identical masses and $n+1$ identical springs, where the extreme ends of the first and last springs are constrained to $y = 0$. Let $y = (y_1, y_2, \ldots, y_n)$ be the vector of transverse positions of the masses (labels ordered in x).

(a) Work to linear order in displacements and write down the differential equation for y_i? [Remember, a spring only exerts a force on mass(es) it is connected to. Draw a picture!]

(b) Write the differential equation for vector y. You likely will need to define a matrix, so give the matrix explicitly.

Comment: There is a large amount of symmetry in this problem, and I hope you find it easy. I expect you will find a tri-diagonal matrix of "Toeplitz" form. The eigenvalues of such a matrix are exactly calculable, and we would find that the frequencies of the normal modes of this system are (in the notation of the text):

$$\omega_k^2 = 2\frac{T}{m\ell}\left(1 + \cos\frac{k\pi}{n+1}\right), \ k = 1, \ldots, n. \tag{2}$$

You may wish to compare with the result in the text for $n = 2$.

As the text notes, we could add more and more masses and approach the limit of a continuous string, and start to describe transverse wave phenomena in continuous media. The pendulum example could be similarly treated to arrive at longitudinal waves (perhaps removing gravity).