Reading: Finish reading Chapter 5 of the textbook, on traveling waves. Read section 6.1, on standing waves.

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20. Here are a few quick (I hope) questions to warm up on:
    (a) We send a positive pulse down our Shive wave machine. The far end is left free to move. Is the reflected pulse positive or negative? What if the far end is clamped – is the reflected pulse positive or negative in this case?
    (b) You have a 100 m length of 50 ohm (that is, the charactersitic impedance is $Z_0 = 50\ \Omega$) coaxial cable and want to know the inductance (L) and capacitance (C) per unit length. So you send a pulse down it and find that it takes 400 ns to get from one end to the other. What are L and C? You may assume the cable is lossless.
    (c) Our Rubens' tube has a standing wave with nodes separated by 1 meter. What is the frequency of the sound wave? Wikipedia says the speed of sound (under typical conditions) is 343 m/s.
    (d) What do the particular (also known as the nonhomogeneous solution) and complementary (also known as the homogeneous solution) solutions correspond to physically [asked by 6 people on question 15]?

21. Your colleague is dangling a rope from the top of Millikan Library at Caltech. You stand at the bottom and pluck it to generate a transverse wave that travels to the top where it reflects and travels back down. How long does it take to get back to you? According to wikipedia, Millikan is 44 m tall (`https://en.wikipedia.org/wiki/List_of_buildings_in_Pasadena,_California`). You may neglect contributions other than tension from gravity. But be careful, is the tension everywhere the same?

22. Let's take our discussion of a lossless transmission line a bit further. Consider the transition between lossless lines with different characteristic impedances shown in the picture below.

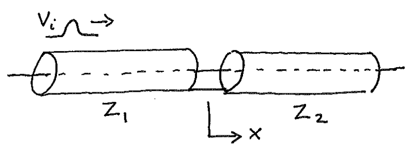

Figure 1: Two transmission lines joined at $x = 0$. The gap is only for visual effect, the lines actually go all the way to $x = 0$ on each side.

We put a voltage pulse in from the left. It travels without loss to the boundary with the Z_2-impedance line, where some portion may be reflected and some portion may keep going down the second line.

(a) What are the boundary conditions that need to be satisfied at the transition ($x = 0$)?
(b) Compute the reflection and transmission coefficients for this transition. Note that the discussion in the text about strings is likely similar to what is needed here. But try to be careful to justify what you do.
(c) Check the limiting cases where $Z_2 \to Z_1$, $Z_2 \to 0$, and $Z_2 \to \infty$. Do these make sense?

23. In quantum mechanics we also talk about waves and a wave equation, called the Schrödinger equation. Consider the problem of a particle of mass m in force-free motion. Classically, it travels in a straight line with constant energy, $E = p^2/2m$, where $p = mv$ is the momentum. The Schrödinger equation for this problem is

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x}, t) = i\hbar\frac{\partial\psi(\mathbf{x}, t)}{\partial t}, \tag{1}$$

where $\hbar \approx 10^{-34}$ kg m^2/s is Planck's constant divided by 2π. This is not quite the form of wave equation we have been looking at. The physical interpretation of the "wave function" ψ (which may be complex) is that the quantity:

$$P(t) = \int_V |\psi(\mathbf{x}, t)|^2 d^3(\mathbf{x}) \tag{2}$$

is the probability to find the particle in volume V at time t.

(a) Solve equation 1 by trying our usual sinusoidal functions (it is good to use exponentials here). Does it look like a wave? Express your solution in terms of a wave vector $\mathbf{k} = (k_x, k_y, k_z)$ and an angular frequency ω (as well as m and $\hbar$). You can let $k \equiv |\mathbf{k}|$ if you like. What is the relation between ω and k?

(b) Using your relation above express the the velocity of the wave in terms of k and m (and $\hbar$).

(c) Now notice that $\hbar k$ has units of momentum and $\hbar\omega$ has units of energy. With this interpretation does the result of part (a) make sense from what you know classically?

(d) Using momentum is mass times velocity and your interpretation of $\hbar k$ as momentum, what is the velocity of the particle? Does this agree with what you derived in part (b)?

Comment: If you have managed to get through this, you have just noticed the distinction between notions of "phase velocity" and "group velocity". We will encounter this again later.
