Ph2a Online @ Caltech **Homework-6**

Vibrations and Waves

Reading: Finish reading Chapter 6 of the textbook, on Standing Waves. Read section 7.1 on Interference and Huygen's Principle.

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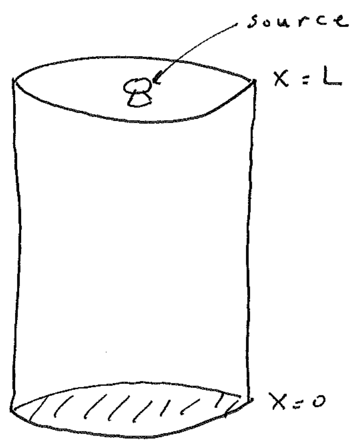
24. Some quick questions:

    (a) A flute and a clarinet are about the same length. What do you expect for the ratio of frequencies? Why? Hint: consider the boundary conditions at the ends.

    (b) A Michelson interferometer has been tuned so that the intensity of light on the detector is zero. So where does the light go? Assume that the optical elements are lossless.

    (c) In class, we drew some pictures of normal modes of a circular drumhead. At some instant in time, we represented upward excursions (from the equilibrium position) by a + sign and downward excursions with a − sign. The nodes were represented with lines. Now do the same for a square drumhead, representing in similar diagrams: (*i*) The lowest frequency mode, (*ii*) The next higher mode that maintains the square symmetry, and (*iii*) the lowest two equal-frequency modes that break the square symmetry.

25. We did a standing wave demonstration of sound waves in a column of air, see the figure below.



The top is open around the source, while the bottom ends at a water surface. We made an observation about the approximate frequencies of the two lowest standing wave modes by listening to the sound level as a function of frequency. Let's see if we can predict what to expect for the ratio of these frequencies.

(a) What are the boundary conditions at $x = 0$ and $x = L$? It may be helpful to remember that sound waves are longitudinal pressure waves.

(b) What do you expect for the ratio of the frequencies of the two lowest standing waves? It may be useful to draw a picture of these waves in the column.

26. We also did a demonstration of standing waves on a taut string with a length of about 2 m. The horizontal string went over a pulley and was weighted down at one end by a weight of around 300 grams under gravity. Both ends are essentially fixed. We did the 10th normal mode and found an amplitude of about 5 cm. What was the energy in this standing wave?

27. Consider a square drumhead, with sides of length $2a \times 2a$. Denote the speed of a wave on this drumhead by v.

    (a) Find the normal modes of the drumhead. You will of course need to state the boundary condition at the edges to do this.

    (b) Give an expression for the general solution to the oscillations of the drumhead, as a superpostion of normal modes.

28. We had an introduction to quantum mechanics in problem 23. Standing waves are a useful concept also here. We saw that for a free particle, the Schrödinger equation effectively says $E = p^2/2m$. We also mentioned in class the deBroglie relation that says that a particle of momentum p can be described as a wave, with wavelength $\lambda = h/p$, where h is Planck's constant. We can generalize this notion to include a potential $V(x)$. In this case, the Schrödinger equation reads

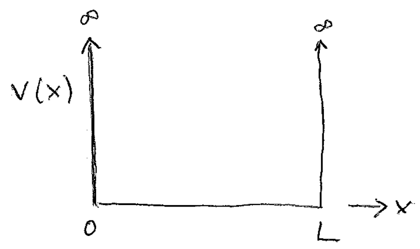
$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t) + V(\mathbf{x})\psi(\mathbf{x},t) = i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t}, \tag{1}$$

where $\hbar \approx 10^{-34}$ kg m^2/s is Planck's constant divided by 2π. We note that if the time dependence is of the form $e^{-i\omega t}$, and we can write $\psi(\mathbf{x},t) = \psi(\mathbf{x})e^{-i\omega t}$, then the Schrödinger equation becomes its "time-independent" form:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x}) + V(\mathbf{x})\psi(\mathbf{x}) = E\psi(\mathbf{x}), \tag{2}$$

where $E \equiv \hbar\omega$. This is an eigenvalue equation for wave function $\psi(\mathbf{x})$, with eigenvalue E, the energy. If our particle is bound by potential V, we will have standing waves.

Let us illustrate this with a very simple nuclear model. A nucleon is bound to a nucleus by the strong force. An approximate model is that the binding potential is zero within the nucleus, and infinite once you get to the nuclear radius. The nucleus is of course three dimensional, but to keep things simple, let's consider a one dimensional approximation, referring to the picture below.



Suppose we wish to determine the eigenvalues, E, that is, the possible energies that a nucleon bound in this "nucleus" can have. We don't need to actually solve the Schrödinger equation to determine the allowed energy levels of a nucleon in this potential. Just like our classical standing waves have quantized frequencies, these energy levels will be quantized.

(a) What are the boundary conditions that our nucleon wave function must satisfy?

(b) Within $x \in (0, L)$ the nucleon has potential energy zero, and hence we can use our sinusoidal free particle solutions in this region, as in problem 24. Using the boundary conditions, determine the possible energy levels of the nucleon.

(c) Use $L = 1$ fm, an approximate nuclear size, and a nucleon mass of 1 GeV/c^2. What is the lowest energy a nucleon can have in the nucleus? Express your answer in MeV, which is a convenient scale for nuclear energies.
