Ph2a Online @ Caltech **Homework-7**

Vibrations and Waves

Reading: Finish reading Chapter 7 of the textbook, on interference and diffraction. Read sections 8.1 and 8.2.

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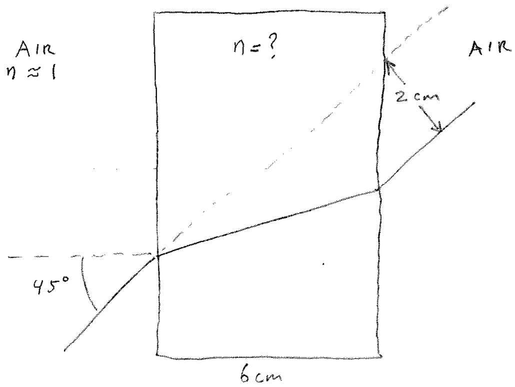
29. Some quick questions:

    (a) In lecture we looked at the refraction of light as it passes through a block of material, roughly as depicted in the figure:

    What is the index of refraction of the material?

    (b) What angular separation can the 200 inch telescope on Mt. Palomar resolve at a wavelength of 2.2 μm? Neglect possible atmospheric effects.

    (c) You shine a beam of unpolarized light at a polarizing filter. If the incident light has intensity I_0, what is the intensity after the polarizer?

    (d) You shine a beam of polarized light on a polarizing filter. The beam is polarizaed along the x direction, with a wave vector in the z direction. You put a polarizer in the beam oriented at angle θ to the x axis (in the $x - y$ plane). If the incident light has intensity I_0, what is the intensity after the polarizer?

30. We discussed the polarization of light and the effect of a "half-wave plate" in class (if you missed class, please see the lecture notes). We consider here the effect of a "quarter wave plate".

    (a) What is the effect on the polarization of a light beam passing through a quarter wave plate? You should consider the effect on an arbitrary complex polarization, that is with the direction of the electric field represented by polarization vector

    $$\mathbf{p} = p_x\hat{\mathbf{x}} + p_y\hat{\mathbf{y}}, \tag{1}$$

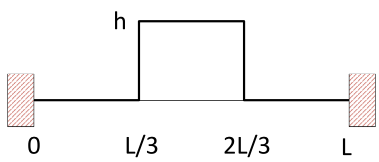
    for a wave traveling in the z direction. Note that p_x and p_y are allowed to be complex here.

    (b) If the incident wave has polarization $\mathbf{p} = \hat{\mathbf{x}}$, what is the outgoing polarization? What if $\mathbf{p} = \hat{\mathbf{y}}$?

    (c) If the incident wave has polarization $\mathbf{p} = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{x}} + \hat{\mathbf{y}}\right)$, what is the outgoing polarization?

    Do you understand your results physically?

31. Consider a taut string of equilibrium length L, fastened at both ends. At time $t = 0$ the middle third is streched into a square pulse shape of height h as shown in the figure below. It is at rest before it is released at $t = 0$.

(a) What is the shape of the string as a function of time? Express your result in terms of h, L, and the wave velocity v (and time, of course). You may assume no energy loss.

(b) Does the string ever return to the initial shape? If so, when?

32. Let's consider how we can use a Michelson interferometer (e.g., Fig. 7.8 in the text) to measure the index of refraction of a gas such as helium. We insert a glass cell into one path of the interferometer and evacuate it. The optical path length of the evacuated region is L. Monochromatic light of wavelength λ is shone into the interferometer. One of the interferometer mirrors is adjusted to give maximum intensity at the detector. Now helium gas is slowly added to the cell until the pressure reaches atmospheric. As this is done, the intensity of the light at the detector will vary due to the slowing velocity in the cell. Careful count is taken of the number of times the intensity varies from maximum to minimum and back to maximum. If $L = 0.1$ m, $\lambda = 633$ nm (from a helium-neon laser), and there are $k = 11$ cycles back to maximum intensity, what is the index of refraction of helium at atmospheric pressure?

33. We have discussed the Young double slit experiment. With many slits, we get a "diffraction grating". Because the diffraction is wavelength-dependent, different wavelengths of light can be dispersed. Let's use our double slit as the simplified prototype for this.

(a) For the double slit experiment, we saw that the intensity, averaged over a cycle, at an angle θ from the slits to the observing point is:

$$\bar{I}(\theta) = 2A^2 \cos^2\left(\frac{ka}{2}\sin\theta\right), \tag{2}$$

where A is the amplitude of the incident plane wave, a is the distance between the slits, and k is the wavenumber of the wave. This equation was derived for a plane wave with normal incidence on the slits. What would the equation be if the angle of incidence (with respect to the normal) is θ_i (instead of zero)?

(b) We see that the intensity will have a series of maxima as a function of angle, and also wavelength. Including the possibility that $\theta_i \neq 0$, determine the values of θ where these maxima occur, as a function of wavelength, spacing a, and angle of incidence.

(c) Consider the first maximum (at non-zero θ). You may assume that θ is small. For a grating spacing of 0.01 mm, what will be the angular separation in degrees between red light ($\lambda \approx 660$ nm) and green light ($\lambda \approx 530$ nm)?